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13. ABSTRACT *(Maximum 200 words)*

New results have been obtained on nonlinear wave equations and plate equations. These results include exact controllability, strong and uniform stabilization, structural damping, quadratic optimal control problem, Riccati equation, robustness with respect to nonlinear uncertainties, on numerical aspects of the operator Riccati Equation. Both boundary control and point control problems have been considered.

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Final Technical Report
Grant No. AFOSR-87-0321
December 1, 1989 through August 31, 1991

INCREASING THE MARGIN OF STABILITY OF ARBITRARILY FINITE MODES OF FLEXIBLE LARGE SPACE STRUCTURES WITH DAMPING

Submitted to:

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Attention:

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February 1991

This final report summarizes the principal investigators' achievements on the research project during the period December 1, 1989 through August 31, 1991 and provides a list of publications under this grant since 1988. These include new results for wave equations and plate equations, linear and nonlinear, on the following problems: exact controllability, strong and uniform stabilization, structural damping, quadratic optimal control problem, Riccati equations, robustness with respect to nonlinear uncertainties, and numerical aspects thereof. Both boundary control and point control problems are considered.

INCREASING THE MARGIN OF STABILITY OF ARBITRARILY FINITE MODES OF FLEXIBLE LARGE SPACE STRUCTURES WITH DAMPING

SECTION I

1. **Premise.** The present technical report covers the period of research December 1, 1989 - August 31, 1991, by the Principal Investigators (P.I.s) and their graduate students. As such, it should be viewed as a successor of the previous report (Report No. UVA/525701/AM90/101) of the same grant No. AFOSR-87-0321A, submitted by the P.I.s on February 1990 to the AFOSR, Control Theory Program, and covering the period September 1, 1988 - November 30, 1989. We find more useful and more informative, however, to provide in Section II of the present document a *complete* list of research publications under Grant AFOSR-87-0321A, appeared since 1988, even though our comments in Section I will refer only to the later grant period December 1, 1989 - August 31, 1991.

2. **Summary of research activities, December 1, 1989 - August 31, 1991.** During this period, research efforts by the P.I.s and their students have centered on boundary control for waves and beams/plate equations with various boundary conditions, as well as corresponding point control problems. More precisely, major themes of research performed under the grant include the following problems:

(i) control problems with optimal quadratic cost over a finite/infinite time horizon and corresponding differential/algebraic Riccati equations

(ii) uniform stabilization in the case of infinite time/algebraic Riccati equations of (i)

(ii) numerical approximations of Riccati equations and corresponding gain operators, optimal solution, optimal control, etc.

(iv) exact controllability/uniform (strong) stabilization of linear and (non-linear) models of wave and plate equations

(v) interior point control for wave and plate models.

Regarding research areas (i) and (ii) an outstanding open problem of physical relevance was solved in [L-T.6], which refer to Riccati Algebraic Equations and related optimal control problem, when the input-solution map is *unbounded.* A highlight in this area was the publication by the P.I.s of the Springer-Verlag Lectures Notes #166 on Riccati equations, Ref [L-T.1].

Regarding area (iii), several numerical approximations works for Riccati equations were completed [L-T.5], [L.2], [L.3]. A graduate student (Erik Henrickson) is presently testing actual numerical computations in his Masters' Thesis.

As to area (iv) of controllability/stabilization, new contributions by the P.I.s are [L-T.12], [L-T.13], [L-T.14] for linear plates, [L.6] through [L.14] for nonlinear wave and plates. Moreover, the research papers by the Ph.D. students listed in § 2.3 of Section II are all in this area. The question regarding the minimal number of boundary controls was settled in particular.

Finally, work on point control, area (v), was completed in [T.8]-[T.10], to show advantages/limitations of this mode of control.

3. **Conclusion:** It is felt that research efforts by the P.I.s and their graduate students offer a useful contribution to Air Force problems and needs in the area of control and stabilization of flexible structures.

SECTION II

List of Work by the Co-Principal Investigators (I. Lasiecka and R. Triggiani) **and their Ph.D. students related to AFOSR Grant 87-0321, appeared since 1988 (or to appear).**

2.1. Numerical approximation of control problems and dynamics. Linear, quadratic

boundary control problems and related Riccati equations

2.1.1 Work by I. Lasiecka and R. Triggiani

Book

[L-T.1] I. Lasiecka and R. Triggiani. *Differential and Algebraic Riccati equations with application to boundary/point control problems: continuous theory and approximation theory,* Volume #164 in the Springer-Verlag Lectures Notes LNCIS series, 1991 (160 pages). (It is an expansion of [L-T.2] below.)

Review paper

[L-T.2] I. Lasiecka and R. Triggiani. Algebraic Riccati equations arising in boundary/point control: a review of theoretical and numerical results. Part I: continuous case. Part II: Approximation theory. In "Perspectives in Control Theory," *Proceedings of the Sielpia Conference,* Poland, 1988, B. Jakubczyk; K. Malanowski; W. Respondek Editors, Birkhauser, 1990, pp. 175-235.

Research papers

[F-L-T.1] F. Flandoli, I. Lasiecka and R. Triggiani. Algebraic Riccati Equations with Non-Smooth Observations Arising in Hyperbolic and Euler-Bernoulli Equations, *Annali di Mathematica Pura et Applicata,* (IV) 1989, Vol. CLIII, pp. 307-382.

[L-T.3] I. Lasiecka and R. Triggiani. Differential Riccati equations with unbounded coefficients: applications to boundary control/boundary observation hyperbolic problem, *J. of Nonlinear Analysis,* Vol. 17, No. 7, (1991), 655-682.

[L-T.4] Lasiecka and R. Triggiani. Riccati differential equations with unbounded coefficients and non-smoothing terminal condition — The case of analytic semigroup, *SIAM J. Mathem Analysis,* to appear.

[L-T.5] I. Lasiecka and R. Triggiani. Numerical approximations of algebraic Riccati equations for abstract systems modelled by analytic semigroups and applications, *Mathematics of Computation,* Vol. 57 No. 196 (1991) 639-662, and *Supplemnt* 513-537.

[L-T.6]I. Lasiecka and R. Triggiani, Riccati equations arising from systems with unbounded input-solution operator: applications to boundary control problems for wave and plate problems, *J. of Nonlinear Analysis,* to appear. Presented at the IFIP Conference, Zurich, Switzerland, Sept. 1991. A concise version will appear in the Springer-Verlag Proceedings.

2.1.2 Work by I. Lasiecka and A. Manitius

[L-M.1] I. Lasiecka and A. Manitius. Differentiability and Convergence Rates of Approximating Semigroups for Retarded Functional Differential Equations, *SIAM J. on Numerical Analysis* , Vol. 25, No. 4, 1988, pp. 883-907.

2.1.3 Work by I. Lasiecka and J. Sokolowski

[L-S.1] I. Lasiecka and J. Sokolowski. Regularity and Strong Convergence of a Variational

Approximation to Nonhomogeneous Dirichlet Hyperbolic Boundary Value Problems, *SIAM Journal of Mathematical Analysis,* Vol. 19, No. 3, 1988 pp. 528-540.

[L-S.2] I. Lasiecka and J. Sokolowski. Semidiscrete Approximations of Hyperbolic Boundary Value Problems with Nonhomogeneous Dirichlet Boundary Conditions, *SIAM Journal Mathematical Analysis* , Vol 20, No. 6, 1989, pp. 1366-1387.

[L-S.3] I. Lasiecka and J. Sokolowski. Regularization and Finite Element Approximations of the wave equation with Dirichlet boundary data (also with P. Neittanmaki), *Numerical Analysis and Mathematical Modelling,* Vol. 24, pp. 329-353 (1990).

2.1.4 Work by I. Lasiecka

[L.1] I. Lasiecka. Finite dimensional approximations of algebraic Riccati equations arising in hyperbolic problems with boundary/point control, Proceedings of NASA-UCLA Workshop on Computational Techniques in identification and control of flexible flight structures, pp. 247-270, (1990) Optimiz. Software Inc.

[L.2] I. Lasiecka. Approximations of solutions to infinite dimensional algebraic Riccati equations with unbounded input operators, *Numerical Functional Analysis and Optimization,* Vol. 11, pp. 303-378 (1990).

[L.3] I. Lasiecka. Convergence rates for the approximations of the solutions to algebraic Riccati equations with unbounded coefficients - the case of analytic semigroups, sumbitted.

2.2 Exact controllability and uniform stabilization of wave/plate equations.

Linear case

2.2.1. Work by I. Lasiecka and R. Triggiani

[L-T.7] I. Lasiecka and R. Triggiani. Exact Boundary Controllability for the Wave Equation with Neumann Boundary Control, *Applied Mathem. and Optimiz.,* Vol. 19, No. 3, (1988), pp. 243-291.

[L-T.8] I. Lasiecka and R. Triggiani. Exact Controllability of the Euler-Bernoulli Equation with Controls in the Dirichlet and Neumann Boundary Conditions: A Non-Conservative Case, *SIAM J. Control and Optimization,* Vol 27 (1989), 330-374.

[L-T.9] I. Lasiecka and R. Triggiani. Exact Controllability of the Euler-Bernoulli Equation with Boundary Controls for Displacement and Moment, *J. Mathem. Anal. & Applic.,* Vol. 146, No. 1, (1990), 1-33.

[L-T.10] I. Lasiecka and R. Triggiani. Uniform stabilization of the wave equations with Dirichlet-feedback control without geometrical conditions, *Appl. Mathem. & Optimiz.,* to appear; also presented at CDC Conference, Tampa, Florida, December 1989. Preliminary version appeared in Springer-Verlag Lectures Notes LNCIS Vol. 147 (1990) 62-108, J. P. Zolesio Editor.

[L-T.11] I. Lasiecka and R. Triggiani. Farther results on exact controllability of the Euler-Bernoulli equation with controls on the Dirichlet and Neumann boundary conditions.

Springer-Verlag Lectures Notes in Control Vol. 147, 1990, 226-235, J. P. Zolesio, Edit.

[L-T.12] I. Lasiecka and R. Triggiani. Exact controllability and uniform stabilization of Kirchhoff plates with boundary control only $\Delta w|_{\Sigma}$ and homogeneous boundary displacement, *J. Differ. Eqts.* Vol. 93 (1991), 62-101. Presented at International Conference on evolution operations and semigroups, University of Delft, The Netherlands, September 1989 and CDC Conference, Tampa, Florida, December 1989. Condensed versions of the paper to appear/appeared in Proceedings.

[L-T.13] I. Lasiecka and R. Triggiani. Optimal regularity exact controllability and uniform stabilization of the Schrodinger equation," *Differential & Integral Eqts.*, to appear.

[L-T.14] I. Lasiecka and R. Triggiani. Exact controllability and uniform stabilization of Euler-Bernoulli equations with only one active control in $\Delta w|_{\Sigma}$, *Boll. Union. Mathem. Ital.* to appear.

2.2.2 Work by I. Lasiecka

[L.4] I. Lasiecka. Controllability of a Viscoelastic Kirhoff Plate, *International Series & Numerical Mathematics*, Vol. 91, Birkhauser, 1989, pp. 237-248.

[L.5] I. Lasiecka. Exact controllability of a plate equation with one control acting as a bending moment, *Lecture Notes in Pure and Applied Mathematics*, Vol. 127, pp. 345-363 (1990) Marcel Dekker, 1990.

2.2.3 Work by S. Chen and I. Lasiecka

[C-L.1] S. Chen and I. Lasiecka. Feedback realization of terminal constrained control problems - applications to feedback exact null-controllability, *J. Optimiz. Theory & Applic.*, to appear.

2.2.4 Work by S. Chen and R. Triggiani

[C-T.1] S. Chen and R. Triggiani. Gevrey class semigroups arising from elastic systems with gentle perturbation, *Proc. Amer. Math. Soc.* Vol. 110 (1990), 401-415.

[C-T.2] S. Chen and R. Triggiani. Characterization of domains of fractional powers of certain operators arising in elastic systems and applications. *J. Differ. Eq.* Vol. 88 (1990) 279-293.

[C-T.3] S. Chen and R. Triggiani. Proof of Extensions of two conjectures on structural damping for elastic systems: the case $\frac{1}{2} \leq \alpha \leq 1$, *Pacific J. of Mathematics* Vol. I 136 (1989), 15-55.

[C-T.4] S. Chen and R. Triggiani. Proof of two conjectures of G. Chen and D. L. Russell on Structural damping for elastic systems: the case $\alpha = \frac{1}{2}$," Lectures Notes in Mathematics, 1354, Springer-Verlag (1988), 234-256.

2.2.5 Work by R. Triggiani

[T.1] R. Triggiani. Wave equation on a bounded domain with boundary dissipation: an operator approach, *Journal of Mathematical Analysis and Application,* Vol. 137 (1989), 438-461.

[T.2] R. Triggiani. Exact boundary controllability of $L_2(\Omega) \times H^{-1}(\Omega)$ of the wave equation with Dirichlet boundary control acting on a portion of the boundary and related problems, *Applied Mathematics and Optimization,* 18 (1988), 241-277.

[T.3] R. Triggiani. Exact controllability of wave and plate equations in the presence of damping, *Lecture Notes in Pure and Applied mathematics* Vol. 119, Marcel Dekker (1989), 377-381.

[T.4] R. Triggiani. Regularity of structurally damped systems with point/boundary control, *J. Mathem. Analysis & Applic.* to appear.

[T.5] R. Triggiani. Lack of exact controllability for wave and plate equations with finitely many boundary controls, *Differential & Integral Equations,* to appear.

[T.6] R. Triggiani. Finite Rank, relatively bounded perturbations of semigroup generators Part III: a sharp result on the lack of uniform stabilization, *Differential and Integral Equations* Vol. 3 (1990) 503-522.

[T.7] R. Triggiani. Constructive steering control functions for linear systems and abstract rank conditions, *J. Optimiz. Theory & Applic,* to appear.

[T.8] R. Triggiani. Interior and boundary regularity with point control. Part I: wave and Euler-Bernoulli equations, *Differential & Integral Equations,* to appear.

[T.9] R. Triggiani. Regularity with point control. Part II: Kirchhoff equations," *J. Diff. Eqts.,* to appear.

[T.10] R. Triggiani. Regularity with point control Part III: Schrodinger equation, *J. Math. An & Appl.,* to appear.

[T.11] R. Triggiani. Counterexamples to some stability questions for dissipative generators, *J. Mathem. Anal. & Applic.,* to appear.

Non Linear case

2.2.6 Work by I. Lasiecka and R. Triggiani

[L-T.15] I. Lasiecka and R. Triggiani. Exact controllability of semi-linear abstract systems with application to waves and plates boundary control problems, *Applied Mathem. & Optimization,* Vol. 23 (1991), 109-154.

2.2.7 Work by I. Lasiecka

[L.6] I. Lasiecka. Boundary stabilization of hyperbolic and parabolic equations with nonlinearly perturbed boundary conditions, *Journal Different. Equat.,* Vol. 75, No. 1, 1988, pp. 53-87.

[L.7] I. Lasiecka. Stabilization of wave and plate-like equations with nonlinear dissipation on the boundary," *Journal Differential Equations,* Vol. 79, No. 2, 1989, pp. 340-381.

[L.8] I. Lasiecka. Stabilization of the semilinear wave equation with viscous damping, *Journal Differential Eq.* Vol. 86 No. 1, (1990), pp. 73-87.

[L.9] I. Lasiecka. Asymptotic behavior of the solutions of the Kirkhoff plate with nonlinear dissipation in the bending moments and shear Forces," *Journal of Applied Mathematics Optimization,* Vol. 21, (1990), pp. 167-189. Short version in Lecture Notes in Control and Information Sciences, Springer Verlag, Vol. 125, "Control of Boundaries" ed. J. Simon, pp. 168-176.

[L.10] I. Lasiecka. Exponential stabilization of hyperbolic systems with nonlinear, unbounded perturbation, *Springer Verlag Lecture Notres,* in Control and Information Sciences, Vol. 154, pp. 102-116, 1991.

[L.11] I. Lasiecka. Stability of wave equation with nonlinear damping in the Dirichlet and Neumann boundary conditions in control of partial differential equations (ed. A. Bermudez). *Lecture Notes in Control and Information Sciences,* Springer Verlag, 1989, pp. 47-65.

[L.12] I. Lasiecka. Exponential local stability of first the order strictly hyperbolic systems with nonlinear perturbations on the boundary, *Lecture Notes in Control and Information Sciences,* Vol. 100, Boundary Control and Boundary Variations (J.P. Zolesio, ed.) (Springer-Verlag: 1988), pp. 212-235.

[L.13] I. Lasiecka. Global uniform decay rates for the solutions to wave equation with nonlinear boundary conditions, *Applicable Analysis,* to appear.

[L.14] I. Lasiecka. Exponential stabilization of hyperbolic systems with nonlinear, unbounded perturbation - Riccati operator approach, *Applicable Analysis,* to appear.

2.3 Work by the P.I.s and their Ph.D. students

a) Ph.D. Granted

a_1)Dept. of Mathematics, University of Florida:
Jerry Bartolomeo, (Ph.D. December 1989; Advisor: R. Triggiani)

$a_{2)}$ Dept. of Applied Mathematics, University of Virginia:
Norman Ourada (Ph.D. July 1990; Advisor: R. Triggiani)

Elizabeth Bradley (Ph.D. July 1991; Advisor: I. Lasiecka)

b) Ph.D. Students (partial list)

Mary Ann Horn (Ph.D. expected summer 1992; Advisor: I. Lasiecka)

Daniel Tataru (Ph.D. expected summer 1992; Advisors: I. Lasiecka and R. Triggiani)

Christine McMillan (Ph.D. expected summer 1993; Advisor: R. Triggiani)

Research papers

J. Bartolomeo and R. Triggiani, Uniform decay rates for Euler-Bernoulli equations with feedback operators in the Dirichlet/Neumann B.C., *SIAM J. Mathemat. Analysis*, Vol. 22 (1991), 46-71.

N. Ourada and R. Triggiani, Uniform Stabilization of the Euler-Bernoulli Equation with Feedback Operator Only in the Neumann Boundary Conditions, *Differential and Integral Equations*, Vol. 4 (1991), 277-292.

E. Bradley and I. Lasiecka, Existence of solutions to an Euler-Bernoulli plate, in *Control & Cybernetics* to appear.

E. Bradley, Uniform stabilization of a nonlinearly perturbed Kirchhoff plate, *Applied Mathematics & Optimization*, to appear.

E. Bradley and I. Lasiecka, Exponential stabilization of a nonlinearly perturbed von Karman plate, *Journal of Nonlinear Analysis: Theory, Methods and* Applications," to appear.

M. A. Horn and I. Lasiecka, The Euler-Bernoulli plate is exactly controllable via bending moments only, to appear in Springer-Verlag Lectures Notes in *Control & Information Sciences*, Proceedings on IFIP Workshop held at Irsee, Germany, April 1990.

M. A. Horn, Exact controllability of the Euler-Bernoulli plate via bending moments only in the space of optimal regularity, *Journal of Mathematical Analysis & Applications*, to appear.

M. A. Horn, Uniform stabilization of the Euler-Bernoulli plate with feedback acting via bending moments only, *Differ. & Integr. Eqts.*, to appear.

C. McMillan, Stabilization of the wave equation with finite range Dirichlet feedback, *J. Mathem. Anal. & Applic.*, to appear.

D. Tataru and I. Lasiecka, Uniform boundary stabilization of semilinear wave equation with nonlinear boundary conditions, *Diff. & Integr. Eqts.*, to appear.

Presently, C. McMillan is writing up a paper on the Differential Riccati equation for plate equations with boundary controls.

Remarks

1) Two of the above Ph.D. students, Mary Ann Horn and Daniel Tataru, have participated in the 1991 SIAM Student Paper Competition (see attached announcement). It was recently announced (see attached letters) that:

(i) Mary Ann Horn has ben selected as one of the three winners of the competition, for her paper "Uniform stabilization of the Euler-Bernoulli plate with feedback acting via bending moments only."

(ii) Daniel Tataru has been selected for honorable mention for his paper "Boundary value problems for first order Hamilton-Jacobi equations."

Both of them have been honored at a special ceremony to be held during the forthcoming ICIAM 91 conference in Washington, DC.

2) To date (February 6, 1992), D. Tataru has received offers for Post-Doctoral/academic positions from the Mathematics Department of the following Institutions: U. of Chicago, M.I.T., Princeton, Harvard, Yale, U. of Minnesota, Purdue U., U. of California S. Diego, Cornell U., Kansas State U., Northwestern U. and has cancelled interviews with some other universities.

3) Mary Ann Horn has been recently awarded:

(i) a two/three years NSF Postdoctoral fellowship in residence at the School of mathematics, University of Minnesota;

(ii) a one-year Post-doctoral fellowship at the Institute of Mathematics and its Applications, University of Minnesota, during the 92-93 year in control theory.

As a result of (i) and (ii), she is giving up pursuing offers for academic positions in some Mathematics Depts. (e.g. U. of Hawaii; Auburn Univ.).

4) Both Mary Ann Horn and Christine McMillan have been selected by the Association of Women in Mathematics (AWM) to participate at a special Workshop by the AWM held during the A.M.S. Annual Meeting in Baltimore, Maryland, January 8-11, 1992, where both have presented their research work.